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SKIN FRICTION AND HEAT TRANSFER
CHARACTERISTICS OF THE COMPRESSIBLE
LAMINAR BOUNDARY LAYER WITH
INJECTION OF A LIGHT, MEDIUM, AND
HEAVY GAS

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SKIN FRICTION AND HEAT TRANSFER CHARACTERISTICS OF THE
COMPRESSIBLE LAMINAR BOUNDARY LAYER WITH INJECTION
OF A LIGHT, MEDIUM, AND HEAVY GAS

by

L. M. Albacete and W. J. Glowacki

ABSTRACT: The effects of the injection of a light, medium, and
heavy gas on the skin friction and heat transfer characteristics
of the laminar boundary layer flow over a flat plate have been
investigated. This was done by a numerical solution of the
governing differential equations for injection rate parameters of
0, -0.1, -0.2; Mach numbers from 0 to 3; and wall-to-stream tem-
perature ratios of 0.25, 0.50, and 1.00. The results show that
the selection of discrete injection systems must represent a
favorable compromise between conflicting factors (such as small
molecular weight and large molecular diameters) which influence
the amount of reduction in skin friction and heat transfer.

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Skin Friction and Heat Transfer Characteristics of the Compressible
Laminar Boundary Layer with Injection of a Light, Medium, and Heavy
Gas

This is the first part of a three-fold investigation into the
effects of foreign gas injection on the laminar boundary layer
heat transfer and stability characteristics. In this report the
boundary layer equations are solved numerically and profiles are
obtained showing the effects of light, medium, and heavy gas
injection. In addition, the effects of the injection on the skin
friction and heat transfer characteristics of these profiles are
examined.

The authors wish to express their gratitude to Dr. John O. Powers
for his advice and encouragement.

                                    E. F. SCHREITER
                                    Captain, USN
                                    Commander

                                    S. M. HASTINGS
                                    By direction

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ILLUSTRATIONS

LIST OF SYMBOLS

C_f	skin friction coefficient
C_p	specific heat at constant pressure
$\mathcal{D}$	binary diffusion coefficient
D_n	variables defined in equation (80)
d_i	molecular diameter or collision cross-section of species i
d_n	degrees of degeneracy in vibration
E	total internal energy of one mole of a perfect gas (c.f., Eq. (44))
Eu	Eucken correction factor for polyatomic gases (c.f., Eq. (42))
E_v	vibrational energy
F	function defined in equation (23)
f	dimensionless stream function
G	function defined in equation (24)
G_{ij}	function defined in equation (33)
H	enthalpy of one mole of a perfect gas (c.f., Eq. (45))
h	enthalpy
k	coefficient of thermal conductivity
Le	Lewis number
k_T	thermal diffusion coefficient
m	molecular weight
n	number of molecules per unit volume
p	pressure
Pr	Prandtl number
Q	total partition function

R	gas constant
Re_x	Reynolds number based on main stream conditions
S	stagnation enthalpy (c.f., Eq. (25))
T	temperature
T_v	vibrational temperature
u	velocity along surface
V	volume
v	velocity normal to surface
w	mass fraction of injected gas
x	distance parallel to surface
Y_n	variable defined by equation (30)
y	distance normal to surface
α	thermal factor
γ	ratio of specific heats
θ_v	vibrational constant (c.f., Eq. (60))
η	transformed y-variable (c.f., Eq. (13))
μ	coefficient of dynamic viscosity
ν	coefficient of kinematic viscosity
ζ	transformed independent variable (c.f., Eq. (22))
ρ	density
Ψ	stream function
Ω^*	reduced collision integral
$\bar{\omega}$	Schmidt number
ω_n	vibrational frequency

SUBSCRIPTS

1	refers to injected gas
2	refers to main stream gas
$\bullet$	refers to free stream
i	i^{th} species
INT	internal degree of freedom
ROT	rotational degree of freedom
TR	translational degree of freedom
VIB	vibrational degree of freedom
W	refers to wall

(Barred symbols denote nondimensional variables defined on page 4).

INTRODUCTION

This report describes the skin friction and heat transfer investigations which were part of a study of the effects of foreign gas injection on the heat transfer and stability characteristics of the laminar boundary layer flow over a flat plate. Specifically, the effect of molecular properties of the injected gas on the skin friction and heat transfer is considered here. The stability characteristics of some of the resulting boundary layer profiles appear in reference (1). In addition, an outline of a numerical method used for a direct numerical solution of the complete system of disturbance equations for the boundary layer stability analysis is given in reference (2).

The effects of injection on the skin friction and heat transfer are of interest in the area of high speed vehicle design, where injection of a foreign gas through the porous wall of the body is recognized as an effective way of alleviating surface heat transfer. The present investigation is based on the analysis of Shen (ref. (3)) who derived a similarity form of the mass transfer boundary layer equations by assuming that the injection velocity varied inversely with the square root of the distance from the leading edge of a flat plate. In his report, Shen did not include any numerical results.

However, Korobkin, in reference (4), did give some results showing that the laminar boundary layer heat transfer characteristics can be appreciably altered depending on the molecular properties of the injected gas. Specifically, his analysis demonstrated that small molecular weight and large molecular diameter of the injected gas are desirable properties for the reduction of skin friction and heat transfer. Furthermore, a large specific heat per unit mass further reduces the heat transfer. In addition, it was shown that whereas it had been indicated that the smaller the molecular weight of a gas, the greater its effectiveness in an injection system, there exists the possibility that the combination of large molecular diameter and a large molar specific heat associated with polyatomic gases may in fact be more significant in reducing skin friction and heat transfer than the low molecular weight of a monatomic gas like helium. (The criterion for the effectiveness of a given gas is normally established as the relative reduction in heat transfer for a given mass-injection rate of the gas.)

In the present investigation, a numerical program for the solution of the governing equations has been used to examine the effects of helium, air, and carbon-tetrachloride injection on the skin friction and heat transfer characteristics of the laminar boundary layer flow over a flat plate. The program includes the effects of equilibrium vibrational excitation, that is, the specific

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heat is assumed to vary with temperature. Results have been obtained
for Mach numbers from 0 to 3; wall-to-stream temperature ratios of
0.25, 0.50, and 1.00; and injection rates of 0, -0.1, and -0.2. The
ranges for the temperature ratio and Mach number are realistic in
terms of a flat plate at Mach numbers up to 3, and for the forward
portion of a blunted re-entry body. As far as the injection rate
is concerned, for a flat plate, separation begins at a value of
this parameter near -0.6 according to our definition. For very
slender bodies (including transverse curvature effects) this value
may be as high as -0.5 before separation takes place. For an
ablation problem, for example, a slightly blunted cone flying at
Mach 15 at 100,000 feet, one might expect values of the injection
rate on the order of -0.4 to -0.6. Dissociation, ionization and
thermal diffusion effects have been neglected. (A study of the
latter appears in reference (5).)

ANALYSIS

1. Basic Equations and Boundary Conditions

We consider the steady two-dimensional flow of a pure gas forced
into the stream through pores of the flat plate. The boundary layer
equations describing the flow of the binary mixture are as follows:

Continuity:
$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{1}$$

Momentum in the x direction:
$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{2}$$

Momentum in the y direction:
$$\frac{\partial p}{\partial y} = 0 \tag{3}$$

Conservation of matter of foreign gas:
$$\rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} = \frac{\partial}{\partial y}\left\{\rho \mathcal{D}\left[\frac{\partial w}{\partial y} + \alpha w(1-w)\frac{\partial \ln T}{\partial y}\right]\right\} \tag{4}$$

Energy:
$$\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{\partial}{\partial y}\left\{k\frac{\partial T}{\partial y} + \rho \mathcal{D}\left[\frac{\partial w}{\partial y}\right.\right.$$
$$\left.\left. + \alpha w(1-w)\frac{\partial \ln T}{\partial y}\right]\left[\frac{k_T}{w(1-w)} + \frac{h_1 - h_2}{RT}\right]\right\} \tag{5}$$

State:
$$p = \rho RT = \text{constant} \tag{6}$$

In these equations, u, v, ρ, T, h, μ, and k represent the velocity components along and normal to the plate, density, temperature, enthalpy, viscosity coefficient, and coefficient of thermal conductivity, respectively. In addition, w is the mass concentration of the foreign gas, defined by:

$$w = \frac{m_1 n_1}{m_1 n_1 + m_2 n_2} \tag{7}$$

where:

m_i = molecular weight of species "i"

n_i = number of molecules of species "i" per unit volume

i = 1 for the injected gas

i = 2 for the main stream gas

Moreover, in equations (1)-(6), the binary diffusion coefficient is given by $\mathcal{D}$, the thermal diffusion coefficient by k_T, and the thermal factor by α, where:

$$k_T = \alpha w(1-w) \tag{8}$$

The gas constant R is for the mixture per unit mass.

Equations (1)-(6) are subject to the following boundary conditions:

$$u(x,0) = 0 \tag{9}$$

$$u(x,\infty) = u_\infty \tag{10}$$

$$T(x,\infty) = T_\infty \tag{11}$$

$$w(x,\infty) = 0 \tag{12}$$

$$v(x,0) \text{ and } T(x,0) \text{ prescribed}$$

It should be pointed out that the temperature of the injected fluid is assumed identical to the temperature of the plate surface. No consideration is given to the source of fluid nor how it attains plate surface temperature.

2. Transformations

As Shen (ref. (3)) has shown, the above equations and boundary conditions will lead to similar solutions for an injection rate varying as $1/\sqrt{x}$ when the following transformations are introduced:

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$$\eta \;=\; \frac{(1/2)\,y}{\sqrt{\dfrac{x\nu_\infty}{u_\infty}}} \tag{13}$$

and the stream function:

$$\Psi \;=\; (\rho_\infty u_\infty \mu_\infty x)^{\frac{1}{2}}\, f(\eta) \tag{14}$$

Under these transformations then:

$$\rho u \;=\; \frac{\partial \Psi}{\partial y} \;=\; \frac{1}{2}\,\rho_\infty u_\infty\,\frac{\partial f}{\partial \eta} \tag{15}$$

$$\rho v \;=\; \frac{-\partial \Psi}{\partial x} \;=\; \frac{1}{2}\left[\frac{\rho_\infty u_\infty \mu_\infty}{x}\right]^{\frac{1}{2}} \left(\eta\,\frac{\partial f}{\partial \eta} - f\right) \tag{16}$$

$$\rho u\,\frac{\partial}{\partial x} + \rho v\,\frac{\partial}{\partial y} \;=\; -\frac{1}{4}\,\rho_\infty u_\infty x^{-1}\, f\,\frac{d}{d\eta} \tag{17}$$

and for any quantity $A(\eta)$ we have:

$$\frac{\partial}{\partial y}\left[A(\eta)\frac{\partial}{\partial y}\right] \;=\; \frac{1}{4}\,\rho_\infty u_\infty x^{-1}\,\frac{d}{d\eta}\left\{A(\eta)\,\frac{d}{d\eta}\right\} \tag{18}$$

Next we introduce nondimensionalized variables (indicated by barred symbols) by dividing all flow variables by their corresponding stream value, except for $\bar{w}$, which is obtained by dividing by the wall value w_w. (Note that w in itself is already dimensionless. The introduction of $\bar{w}$ is for the convenience of certain special cases.)

Using then equations (15)-(18), in terms of the dimensionless variables, equations (1)-(6) are reduced to the following ordinary equations obtained by Shen:

$$\frac{d}{d\zeta}\left\{\bar{\rho}\bar{\mu}\,\frac{d^2 f}{d\zeta^2}\right\} + f\,\frac{d^2 f}{d\zeta^2} \;=\; 0 \tag{19}$$

$$\frac{d}{d\zeta}\left\{\bar{\rho}\bar{\mu}\,\frac{d\bar{w}}{d\zeta}\right\} + f\,\frac{d\bar{w}}{d\zeta} \;=\; -\frac{d}{d\zeta}\left[\bar{\rho}\bar{\mu}F\right] \tag{20}$$

$$\frac{d}{d\zeta}\left[\bar{\rho}\bar{\mu}\,\frac{d\bar{s}}{d\zeta}\right] + f\,\frac{d\bar{s}}{d\zeta} \;=\; -\frac{d}{d\zeta}\left[\bar{\rho}\bar{\mu}G\right] \tag{21}$$

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where:

$$\xi = \text{new independent variable} = \int_0^\eta \bar{\rho}\,d\eta = \int_0^\eta \frac{d\eta}{RT} \tag{22}$$

$$F = \left[\frac{1}{\bar{\omega}} - 1\right] \frac{d\bar{w}}{d\xi} + \alpha \frac{\bar{w}}{\bar{\omega}} (1 - w_w \bar{w}) \frac{d}{d\xi} \ell nT \tag{23}$$

$$G = \frac{1}{\left[1 + \frac{\gamma-1}{2} M^2\right]_\infty} \left\{ \frac{1}{Pr} \bar{C}_\rho \frac{d\bar{T}}{d\xi} - \frac{d\bar{h}}{d\xi} + \right.$$

$$\left. + \frac{w_w}{\bar{\omega}} \bar{T} \left[\frac{\alpha R + C_{p_1} - C_{p_2}}{C_{p_2}}\right]\left[\frac{d\bar{w}}{d\xi} + \alpha \bar{w}(1 - w_w \bar{w}) \frac{d}{d\xi} \ell nT\right]\right\} \tag{24}$$

$$S = h + \frac{1}{2} u^2 = \text{stagnation enthalpy} \tag{25}$$

$$Pr = \frac{\mu C_p}{k} = \text{Prandtl number of local mixture}$$

$$\bar{\omega} = \frac{\mu}{\rho \mathcal{D}} = \text{Schmidt number of local mixture}$$

$$\gamma = C_p/C_v$$

In deriving equation (24), Shen assumed C_p to be concentration-dependent $[C_p(w)]$ but independent of temperature. In order to include temperature dependence in C_p, the following equation for G is used in the present report instead of (24):

$$G = \frac{h_\infty}{S_\infty} \left\{\left(\frac{1}{Pr} - 1\right) \frac{d\bar{h}}{d\xi} + \left(\frac{Le-1}{Pr}\right) w_w \left(\bar{h}_1 - \bar{h}_2\right) \frac{d\bar{w}}{d\xi}\right\} \tag{26}$$

where $Le = Pr/\bar{\omega} = $ Lewis number. This formula reduces to (24) when $\alpha = 0$ and C_p is assumed independent of temperature.

Now, under the previous transformations, the boundary conditions (9)-(12) become:

$$\text{(i)} \quad \frac{df}{d\xi}\bigg|_{\xi \to \infty} = 2 \qquad\qquad \text{(ii)} \quad \frac{df}{d\xi}\bigg|_{\xi=0} = 0$$

$$\text{(iii)} \quad f(0) = -2\bar{\rho}\bar{v}\bigg|_{\xi=0} \sqrt{Re_x} \tag{27}$$

(i) $\bar{w}(0) = 1$ (ii) $\bar{w}(\infty) = 0$ (28)

(i) $\bar{S}(0) = \bar{S}_w = \text{constant}$ (ii) $\bar{S}(\infty) = 1$ (29)

where Re_x is the Reynolds number based on x and the main stream conditions, $= u_\infty x / \nu_\infty$.

The equations describing the problem, solved numerically in the present investigation, are equations (19)-(21), with boundary conditions (27)-(29).

3. Expressions for Transport Properties

A. Binary Diffusion Coefficient

From reference (6), the expression for the binary diffusion coefficient in practical units is:

$$\mathcal{D}_{ij} = 262.8 \times 10^{-5} \frac{[T^3(m_i + m_j)/2\, m_i m_j]^{\frac{1}{2}}}{p d_{ij}^2\, \Omega^{(1,1)*}} \qquad (30)$$

where:

$\mathcal{D}_{ij}$ = diffusion coefficient, cm^2/sec

d_{ij} = $\frac{1}{2}(d_i + d_j)$, angstroms. (the symbol d_i stands for molecular diameter or collision cross-section of species i)

p = pressure, atm.

T = temperature, °K

$\Omega^{(1,1)*}$ = "reduced" collision integral = 1 for rigid sphere model as used in the present investigation

B. Coefficient of Viscosity

For a pure gas of species i, reference (6) gives:

$$\mu_i = 266.93 \times 10^{-7} \frac{\sqrt{m_i T}}{d_i^2\, \Omega^{(2,2)*}} \qquad (31)$$

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(for a rigid sphere model $\Omega^{(2,2)*} = 1$)

where:

μ_i = viscosity of species i, g/cm - sec

m_i = molecular weight of species i

d_i = collision diameter, angstroms

T = temperature, $^{\circ}$K

And for a binary mixture of gases, we have:

$$\mu = \frac{\mu_i}{1 + G_{ij}\left(\frac{n_j}{n_i}\right)} + \frac{\mu_j}{1 + G_{ji}\left(\frac{n_i}{n_j}\right)} \tag{32}$$

where:

$$G_{ij} = \frac{1}{\sqrt{2}} \left(\frac{d_i + d_j}{2d_i}\right)^2 \left(1 + \frac{m_i}{m_j}\right)^{\frac{1}{2}} \tag{33}$$

for $T_1 = T_2$.

Nondimensionalizing (31) we have:

$$\bar{\mu}_1 = \frac{\mu_1}{\mu_\infty} = \frac{\sqrt{m_1 T}}{d_1^2} \cdot \frac{d_2^2}{\sqrt{m_2 T_\infty}} = \left(\frac{d_2}{d_1}\right)^2 \sqrt{\frac{m_1}{m_2}} \sqrt{\bar{T}} \tag{34}$$

$$\bar{\mu}_2 = \frac{\mu_2}{\mu_\infty} = \frac{\sqrt{m_2 T}}{d_2^2} \cdot \frac{d_2^2}{\sqrt{m_2 T_\infty}} = \sqrt{\bar{T}} \tag{35}$$

where the subscripts 1 and 2 indicate injected gas and main stream gas respectively.

Finally, nondimensionalizing (32) we have:

$$\bar{\mu} = \frac{\mu}{\mu_\infty} = \frac{\bar{\mu}_1}{1 + G_{12}\left(\frac{n_2}{n_1}\right)} + \frac{\bar{\mu}_2}{1 + G_{21}\left(\frac{n_1}{n_2}\right)} \tag{36}$$

C. Thermal Conductivity

For a monatomic gas of species i, the Chapman-Enskog Theory yields:

$$k_i = \frac{15}{4} R_i \mu_i \tag{37}$$

where:

k_i = thermal conductivity, cal/cm-sec-°K

μ_i = viscosity given by equation (31)

R_i = gas constant for species i

Substitution of (31) into (37) yields:

$$k_i = 1.9889 \times 10^{-4} \frac{1}{m_i} \frac{\sqrt{m_i}}{d_i^2} \sqrt{T} \tag{38}$$

Nondimensionalizing (38) we have:

$$\bar{k}_1 = \frac{k_1}{k_\infty} = \sqrt{\frac{m_2}{m_1}} \left(\frac{d_2}{d_1}\right)^2 \sqrt{\bar{T}} \tag{39}$$

$$\bar{k}_2 = \frac{k_2}{k_\infty} = \sqrt{\bar{T}} \tag{40}$$

For a mixture of monatomic gases, reference (6) has:

$$\bar{k} = \frac{k}{k_\infty} = \frac{\bar{k}_1}{1 + (1.065) \, G_{12}\frac{n_2}{n_1}} + \frac{\bar{k}_2}{1 + (1.065) \, G_{21}\frac{n_1}{n_2}} \tag{41}$$

The above formulas can be extended to polyatomic gases, as indicated by Dorrance in reference (6). The correction factor Eu (the Eucken correction factor) is:

$$Eu = 1 - L' + \frac{2}{5} L' \frac{C_p}{R_i}$$

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where:

k' = corrected thermal conductivity for polyatomic gases

k = thermal conductivity calculated from monatomic considerations

$$L' = \frac{\rho \mathcal{D}_i (5/2) R_i}{k}$$

$R = R_i m_i$ = universal gas constant

L' can be evaluated using the Chapman-Enskog theory for monatomic molecules. As indicated in reference (6), a good value is $L' \cong 0.885$. Using this value, the correction factor becomes:

$$Eu = k'/k = 0.115 + 0.354\, C_p/R_i \tag{42}$$

Furthermore, the Chapman-Enskog expansion for the thermal conductivity of a mixture of monatomic gases can be modified making use of the Eucken factor to yield an expression for the thermal conductivity of polyatomic gases. The equation is (for a binary mixture):

$$k = \frac{k_1'}{1 + 1.065\, G_{12}\frac{n_2}{n_1}} + \frac{k_2'}{1 + 1.065\, G_{21}\frac{n_1}{n_2}}$$

$$= \frac{k_1\, Eu_1}{1 + 1.065\, G_{12}\frac{n_2}{n_1}} + \frac{k_2\, Eu_2}{1 + 1.065\, G_{21}\frac{n_1}{n_2}}$$

Nondimensionalizing, we obtain:

$$\bar{k} = k/k_\infty = \frac{\bar{k}_1\, Eu_1}{1 + 1.065\, G_{12}\frac{n_2}{n_1}} + \frac{\bar{k}_2\, Eu_2}{1 + 1.065\, G_{21}\frac{n_1}{n_2}} \tag{43}$$

where $\bar{k}_1$ and $\bar{k}_2$ are given by (39) and (40), respectively, and Eu by (42). In addition, G_{ij} is given by (33) for $T_1 = T_2$.

4. Enthalpy and Specific Heat

The total internal energy, E, of one mole of a perfect gas, including translational as well as internal degrees of freedom, may be expressed by the following equation as indicated by Herzberg in reference (7):

$$E_i = E_i^o + R_i T^2 \frac{d}{dT} (\ln Q_i) \qquad (44)$$

where the subscript i indicates that we are talking about a given gas of ith species. In this equation, Q is the total partition function, E^o the energy at absolute zero temperature, T the absolute temperature, and R the gas constant.

Now, the enthalpy, H, of one mole of a perfect gas is the sum of the total internal energy, E, and the external energy pv = RT, such that:

$$H_i = E_i^o + R_i T + R_i T^2 \frac{d}{dT} (\ln Q_i) \qquad (45)$$

Finally, the specific heat at constant pressure is given by:

$$C_{p_i} = \frac{dH_i}{dT} = R_i + R_i \frac{d}{dT} \left[T^2 \frac{d}{dT} (\ln Q) \right] \qquad (46)$$

Now, Q can always be written as the product of the translational and internal partition function Q_{TR_i} and Q_{INT}. Therefore, since Q occurs in (45) and (46) only as $\ln Q$, then the enthalpy and specific heat are the sums of a "translational" and an "internal" term. In particular, for:

$$Q_{TR} = 1.879 \times 10^{20} V m^{3/2} T^{3/2}$$

(where V is the volume considered, m the chemical atomic or molecular weight, T the temperature...for details see Herzberg, reference (7)), we can write:

$$H_i = E_i^o + \frac{5}{2} R_i T + H_{INT} \qquad (47)$$

$$C_{p_i} = \frac{5}{2} + C_{p_{INT_i}} \qquad (48)$$

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where:

$$H_{INT_i} = R_i T^2 \frac{d}{dT} (\ell n Q_{INT})$$

$$C_{p_{INT_i}} = R_i \frac{d}{dT} \left[T^2 \frac{d \; \ell n Q_{INT}}{dT} \right] \tag{49}$$

Moreover, Q_{INT} can be written as the product of a number of factors if certain approximations are made. In particular, if the interaction of vibration and rotation is neglected, both H_{INT} and $C_{p_{INT}}$ can be written as the sums of a rotational and vibrational term such that:

$$H_{INT_i} = H_{ROT_i} + H_{VIB_i} \tag{50}$$

$$C_{p_{INT_i}} = C_{p_{ROT_i}} + C_{p_{VIB_i}} \tag{51}$$

where:

$$H_{ROT_i} = R_i T^2 \frac{d}{dT} (\ell n Q_{ROT}) \tag{52}$$

$$C_{p_{ROT_i}} = R_i \frac{d}{dT} \left[T^2 \frac{d}{dT} (\ell n Q_{ROT} \right] \tag{53}$$

$$H_{VIB_i} = R_i T^2 \frac{d}{dT} (\ell n Q_{VIB}) \tag{54}$$

$$C_{p_{VIB_i}} = R_i \frac{d}{dT} \left[T^2 \frac{d}{dT} (\ell n Q_{VIB}) \right] \tag{55}$$

A. Monatomic Molecules: Helium

For monatomic molecules, in which only the translational degrees of freedom exist, we have from (43):

$$C_{p_i} = \frac{5}{2} R_i \tag{56}$$

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and from (47), for $E_i^0 = 0$ we have:

$$H_i = \frac{5}{2} R_i T = C_{p_i} T \tag{57}$$

For helium $R_i = 0.4967$ cal/gm-$^\circ$K such that $C_{p_{i_{He}}} = 1.24$ cal/gm-$^\circ$K. These are the values used in the present investigation.

B. Diatomic Molecules

(1) Vibrational Excitation Neglected

The effects of translation offer nothing new, being the same for diatomic as well as for monatomic gases (as well as for polyatomic).

The equations for Q_{ROT} for molecules without free or hindered internal rotations are given by Herzberg in reference (7). From the equation for the case of diatomic (and linear polyatomic) molecules we have, after substituting in (52) and (53):

$$H_{ROT_i} = R_i T$$

$$C_{p_{ROT_i}} = R_i$$

Therefore, neglecting the vibrational contribution we have, for diatomic molecules:

$$H_i = \frac{7}{2} R_i T = C_{p_i} T$$

$$C_{p_i} = \frac{7}{2} R_i$$

For air, then, considered as a "mythical diatomic molecule" we have $R_i = 0.0685$ cal/gm-$^\circ$K and thus $C_{p_{i_{AIR}}} = 0.24$ cal/gm-$^\circ$K. These are the values used in the present investigation.

(2) Vibrational Excitation Included

For diatomic molecules with vibrational excitation included, the H_{INT} term in equation (47) contains the contribution due to rotation, $R_i T$ as before, plus now a vibrational term coming

from Q_{VIB} which we shall denote as E_v. This "vibrational energy" can be written as (see reference (8)):

$$E_{v_i} = R_i \frac{\theta_{v_i}}{e^{\theta_{v_i}/T_v} - 1} \tag{60}$$

where:

$$\theta_v = \frac{\text{Planck's constant x vibration frequency}}{\text{Boltzmann's constant}}$$

= characteristic vibrational temperature for given gas

T_v = vibrational temperature (for equilibrium $T_v = T$)

Hence, the expression for the enthalpy is, assuming equilibrium:

$$H_i = \frac{7}{2} R_i T + E_v (T) \tag{61}$$

and the specific heat at constant pressure is:

$$c_{p_i} (T) = \frac{dH_i}{dT} = \frac{7}{2} R_i + c_{p_{i_{VIB}}} = \frac{7}{2} R_i + E_v' (T) \tag{62}$$

where the prime denotes differentiation with respect to T. In particular, from equation (60),

$$E_v' (T) = R_i \left\{ \frac{\theta_{v_i}/2T}{\sinh \theta_{v_i}/2T} \right\} \tag{63}$$

In the present investigation, as in reference (9), air is treated as a "mythical diatomic" gas with R_i = 0.06851 cal/gm-°K and θ_v = 3076°K.

C. Polyatomic Molecules

In this case, the effects of translation offer nothing new, being the same as for the monatomic and diatomic gases. For linear molecules, the treatment of rotation given for diatomic gases is applicable. For nonlinear molecules, such as CCL_4 and CH_4, the effect of rotation adds $(3/2)R_iT$ to the enthalpy, such that:

$$H_{i_{ROT}} = \frac{3}{2} R_i T = c_{p_{ROT}} T \tag{64}$$

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In particular, from reference (7):

$$\omega_1 = 2914.2 \text{ cm}^{-1} \qquad \omega_2 = 1526 \text{ cm}^{-1}$$

$$d_1 = 1 \qquad d_2 = 2$$

$$\omega_3 = 3032.3 \text{ cm}^{-1} \qquad \omega_4 = 1306.2 \text{ cm}^{-1}$$

$$d_3 = 3 \qquad d_4 = 3$$

$$R_{CH_4} = 0.1242 \text{ cal/cm-}^\circ\text{K}$$

(molecular weight of $CH_4 = 16$)

The specific heat at constant pressure is given by:

$$C_{p_{i_{VIB}}} (T) = E_V' (T) = R_i \sum_n d_n \left\{ \frac{\theta_{v_n}/2T}{\sinh (\theta_{v_n}/T)} \right\}^2 \tag{68}$$

which is, except for the summation sign and the factor d_n, the same as (62) and (63) for diatomic molecules. That is, the vibrational contribution to the specific heat can also be represented as the sum of smaller "specific heats" corresponding to the fundamental modes of vibration multiplied by the degrees of degeneracy in each.

Finally, the total enthalpy for these polyatomic gases is then:

$$H_i = \underset{\text{(translation)}}{5/2\ R_i T} + \underset{\text{(rotation)}}{3/2\ R_i T} + \underset{\substack{\text{(vibration,} \\ \text{given by (66))}}}{\sum_n E_{v_n}}$$

or

$$H_i = 4 R_i T + \sum_n E_{v_n} \tag{69}$$

The specific heat at constant pressure is:

$$C_{p_i} = \underset{\text{(translation)}}{5/2\ R_i} + \underset{\text{(rotation)}}{3/2\ R_i} + \underset{\substack{\text{(vibration,} \\ \text{given by (68))}}}{C_{p_{i_{VIB}}} (T)} \tag{70}$$

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If we denote by $C_{p_{T+R}}$ the contributions to the specific heat which are independent of temperature and thus constant for a given gas, i.e., translation and rotation, then we have the more convenient expressions:

$$H_i = C_{p_{T+R}} T + E_v (T) \tag{71}$$

$$C_{p_i} = C_{p_{T+R}} + E_v' (T) \tag{72}$$

which apply to all gases (with the values of $C_{p_{T+R}}$ and E_v (T) being different for each, of course).

Suppose all the vibrational modes of a given gas are excited, say CCL_4 (as in reference (4)). Then for each vibrational degree of freedom the contribution to $C_{p_{VIB}}$ is R_i. Therefore, for the 4.7 vibrational degrees of freedom in CCL_4 ($R_{CCL_4} = .0129$ cal/gm-$^\circ$K) we have: $C_{p_{VIB}} = 4.7 (.0129) = .0604$ cal/gm-$^\circ$K. The total specific heat at constant pressure is then:

$$C_{p_{CCL_4}} = 4R_{CCL_4} + C_{p_{VIB}}$$
$$\text{(translation + rotation)}$$
$$= .0516 + .0604$$
$$= .1120 \quad \text{cal/gm-}^\circ\text{K}$$

The specific heat at constant pressure of the mixture is either concentration-dependent alone (vibrational excitation neglected), or both concentration and temperature-dependent (vibrational excitation included). In any case,

$$C_p = w C_{p_1} + (1-w) C_{p_2} \tag{73}$$

where C_{p_1} and C_{p_2} are either $C_{p_{T+R}}$ or $C_{p_{T+R}} + C_{p_{VIB}}$ depending on whether vibrational excitation is neglected or included.

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Nondimensionalizing (73), we have:

$$\overline{C_p} = \frac{w\, C_{p_1} + (1-w)\, C_{p_2}}{C_{p_\infty}} \tag{74}$$

where in the present investigation we have considered

$$C_{p_\infty} = (C_{p_{T+R}})_2\, T_\infty \tag{75}$$

always. The same reasoning applies for the enthalpy of the mixture:

$$H = w\, H_1 + (1-w)\, H_2$$

$$\overline{H} = \frac{w\, H_1 + (1-w)\, H_2}{(C_{p_{T+R}})_2\, T_\infty} \tag{76}$$

where H_1 and H_2 either neglect or include vibrational excitation.

5. Energy Transfer Across Interface

As indicated by Korobkin in reference (4), the "energy flux across the interface" (i.e., the energy which will increase the body temperature) can be expressed as:

$$(-E)_w = \left[k\, \frac{\partial T}{\partial y} - h_1 \rho v \right]_w \tag{78}$$

Using the transformations outlined previously, this can be written in the dimensionless form:

$$(-\overline{E})_w = \overline{k}_w\, \overline{\rho}_w\, \frac{d\overline{T}}{d\xi}\bigg|_w - \overline{T}_w \left| f(0) \right| Pr_\infty\, \frac{C_{p_1}}{C_{p_2}} \tag{79}$$

That is, the surface heat transfer is seen to be divided into contributions coming from a "temperature gradient" term and a "concentration term."

Now, it is desired to reduce $(-\overline{E})_w$. Evidently, this can be done by:

a. Decreasing $\overline{k}_w$. This may be accomplished, for example, by injecting a large molecular diameter gas, since $\overline{k}_w \sim \frac{1}{d_1^2}$.

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b. Increasing $\bar{T}_w$. This will increase the magnitude of the negative term.

c. Increasing C_{p_1}. Since $C_{p_1} = \dfrac{(C_p/\text{mole})}{(\text{molecular weight})_1}$,

this may be accomplished by either having an injectant of small molecular weight, or one with a high specific heat per mole (say, as a result of vibrational excitation).

Evidently, the relative efficiency of any of these actions depends on the importance of the term in which the affected parameter appears. For example, if the injection is going to be small (small $f(0)$) then perhaps the heat reduction efforts should be concentrated on the first term of (79), i.e., on reducing $\bar{K}_w$, whereas if the injection is going to be considerable, the second term might be more important.

In addition, all of the factors, (a), (b), (c), do not necessarily go together. And so one might have a large molecular diameter gas but with a high molecular weight (an effect which may in turn be corrected by a high specific heat per unit mass), and so on.

In sum, the ideal situation is to have a favorable combination of all these factors. Reflecting this fact, we will see in the present report that at some wall-to-free stream temperature ratios, the light gas (helium in our case) is the most effective coolant, whereas at other temperature ratios, the heavier gas is more advantageous.

6. Numerical Solution of the Equations

Equations (19)-(21), subject to the boundary conditions (27)-(29), have been solved on an IBM 7090 computer using the program known as F NOL 2 described in reference (10). This program is a subroutine which employs the Runge-Kutta and Adams-Moulton fourth order techniques for the integration of a system of ordinary differential equations (up to 30).

Mathematically, the problem consists in solving three ordinary differential equations for the three functions $f(\xi)$, $\bar{s}(\xi)$, and $\bar{w}(\xi)$. These equations are linked. In order to use F NOL 2 , these three equations are reduced to a system of seven first order ordinary differential equations, given by:

$$D_1 = \frac{dY_1}{d\xi} = \frac{-f}{\overline{\rho\mu}} Y_1 \quad \text{where} \quad Y_1 = \overline{\rho\mu} \frac{d^2f}{d\xi^2} \tag{30a}$$

$$D_2 = \frac{dY_2}{d\xi} = \frac{1}{\overline{\rho\mu}} Y_1 \quad \text{where} \quad Y_2 = \frac{df}{d\xi} \tag{80b}$$

$$D_3 = \frac{dY_3}{d\xi} = Y_2 \quad \text{where} \quad Y_3 = f \tag{80c}$$

$$D_4 = \frac{dY_4}{d\xi} = -\frac{f\overline{\omega}}{\overline{\rho\mu}} Y_4 \quad \text{where} \quad Y_4 = \frac{\overline{\rho\mu}}{\overline{\omega}} \frac{d\overline{w}}{d\xi} \tag{80d}$$

$$D_5 = \frac{dY_5}{d\xi} = \frac{\overline{\omega}}{\overline{\rho\mu}} Y_4 \quad \text{where} \quad Y_5 = \overline{w} \tag{80e}$$

$$D_6 = \frac{dY_6}{d\xi} = -f\frac{d\overline{s}}{d\xi} \quad \text{where} \quad Y_6 = \overline{\rho\mu}\left(\frac{d\overline{s}}{d\xi} + G\right) \tag{80f}$$

$$D_7 = \frac{dY_7}{d\xi} = \frac{d\overline{s}}{d\xi} \quad \text{where} \quad Y_7 = \overline{s} \tag{80g}$$

Equations (80a), (80b), and (80c) come from (19); equations (80d), and (80e) come from (20); and equations (80f) and (80g) from (21).

Now, the following boundary conditions must be given for solution of these equations: $f(0)$; $f'(0)$, $f''(0)$, $\overline{s}(0)$, $\overline{s}'(0)$, $\overline{w}(0)$, $\overline{w}'(0)$ where the prime denotes differentiation with respect to ξ. Unfortunately, not all of these conditions are actually given. In particular, we are missing $f''(0)$, $\overline{s}'(0)$ and $\overline{w}'(0)$. But we do have $f'(\infty)$, $\overline{s}(\infty)$, and $\overline{w}(\infty)$. Therefore, a scheme must be devised to obtain the missing boundary conditions which will result in given conditions at $\xi = \infty$. Once they are obtained, the differential equations may be solved numerically using FNOL2.

The procedure to obtain the required boundary conditions used in the present investigation is based on the one employed in reference (11). Stated abstractly, we have three functions (f,g,h) of three variables (x,y,z) and we wish to find the point at which they are zero. In particular, if

$$f(x,y,z) = f'(\infty) - 2$$

$$g(x,y,z) = \overline{s}(\infty) - 1$$

$$h(x,y,z) = \overline{w}(\infty)$$

where $x = f''(0)$, $y = \overline{s}'(0)$ and $z = \overline{w}(0)$, we ask for what values of x, y and z are f=g=h=0 (i.e., $f'(\infty) = 2$, $\overline{s}'(\infty) = 1$, $\overline{w}(\infty) = 0$).

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Such a problem can be solved using the Newton-Raphson iteration method. The iteration equations are, in matrix notation:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{n+1} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{n} - \begin{pmatrix} \dfrac{\partial f}{\partial x} & \dfrac{\partial f}{\partial y} & \dfrac{\partial f}{\partial z} \\[6pt] \dfrac{\partial g}{\partial x} & \dfrac{\partial g}{\partial y} & \dfrac{\partial g}{\partial z} \\[6pt] \dfrac{\partial h}{\partial x} & \dfrac{\partial h}{\partial y} & \dfrac{\partial h}{\partial z} \end{pmatrix}^{-1} \begin{pmatrix} f \\ g \\ h \end{pmatrix}_{n} \tag{81}$$

This iteration would then be accomplished as follows:

(a) Assume first trial values of $f''(0)$, $\bar{s}'(0)$ and $\bar{w}(0)$. (These are x_0, y_0, z_0, respectively.) Integrate the differential equations (30) using F N O L 2 obtaining values of $f'(\infty)$, $s(\infty)$, $w(\infty)$. These are f_0, g_0, h_0, respectively.

(b) Three more complete integrations are made, each differing from that of (a) by a small perturbation of one of the initial conditions $f''(0)$, $\bar{s}'(0)$, $\bar{w}(0)$, i.e., (x, y, z). In each case, values of $f'(\infty)$, $s(\infty)$ and $w(\infty)$ are obtained.

(c) From the results of (a) and (b) approximate values of $\dfrac{\partial f}{\partial x}$, etc. are computed in an obvious manner.

(d) The matrix of partial derivatives is inverted and new values of x, y, z are computed from (31).

(e) The process stops when f, g, h are as close to zero as specified (depending on the accuracy desired). The last x, y, z are the missing values of $f''(0)$, $\bar{s}'(0)$ and $\bar{w}(0)$ which meet the known boundary conditions at infinity, and the integration with these values is the solution to (80) with the appropriate boundary conditions.

Additional details are given in reference (9). The interval of integration used in the present investigation was $\Delta\zeta = 0.1$ for the case of no vibrational excitation, and $\Delta\zeta = 0.01$ with vibrational effects included.

RESULTS AND DISCUSSION

As explained in the Introduction, the main purpose of this part of our investigation was to obtain boundary layer profiles for evaluation of their stability characteristics by the method described in reference (1). In addition, once the profiles were obtained, the effect of foreign gas injection on their skin friction and heat transfer characteristics was examined.

Figure (1) presents the effects of injection on the skin friction coefficient at a representative Mach number (0.70) and wall-to-stream temperature ratio (0.50). The expression for $C_f\sqrt{Re_x}$ is obtained from:

$$C_f\sqrt{Re_x} = \frac{\mu_\infty [\partial u/\partial y]_0}{\frac{1}{2}\rho_\infty u_\infty^2}\sqrt{Re_x}$$

which becomes, after the transformations have been applied:

$$C_f\sqrt{Re_x} = \left[\frac{1}{2}\,\overline{\rho\mu}\,\frac{d^2 f}{d\zeta^2}\right]_w \tag{32}$$

We see in the figure that helium, the lightest gas, gives the sharpest reduction in the skin friction coefficient as the injection is increased. This is due to the effect of molecular weight of the injected gas on the quantity $\overline{\rho\mu}$. In figure (2) the injection rate is held constant at the same Mach number, and the wall-to-stream temperature ratio is varied. Again helium appears most favorable. Finally, in figure (3) the Mach number is varied and the injection rate and temperature ratio are held constant with similar results. So we see that for these ranges of Mach number, temperature ratio, and injection rate the effect of injection on the skin friction is directly related to molecular weight.

The heat transfer results are not as straightforward as in the case of skin friction. Consider figure (4) which shows the effects of wall-to-stream temperature ratio on the ratio of Stanton number with injection to Stanton number with no injection. The figure might best be understood by looking at equation (79). We see that at temperature ratios greater than .36, helium, the lightest gas, appears as the most effective coolant. This is because the value of $\overline{T}_w$ is high enough to make important the second term in (79). Therefore, the reduction in heat transfer achieved by injecting a small molecular weight gas (which further increases this term) is further enhanced by the large value of $\overline{T}_w$. However, at lower values of $\overline{T}_w$, this term is no longer important, and effort should be concentrated in decreasing the first term. This may be accomplished by injecting a gas with large molecular diameter, and thus decreasing $\overline{k}_w$. The figure indeed shows that at lower temperature ratios, the heavier gases are more effective than helium in reducing heat transfer.

In figure (5), the results are plotted versus Mach number for various temperature ratios and an injection rate of -0.1. In addition to confirming again the preceding analysis, the figure shows that the results for a given $\overline{T}_w$ do not vary much with Mach number (except $\overline{T}_w = 0.50$ for helium injection). In fact, a correlation has been suggested by Gross and others (ref. (12)) to collapse all the data at different Mach numbers and temperature ratios into one curve. This is done by plotting the Stanton number ratio versus one half the injection
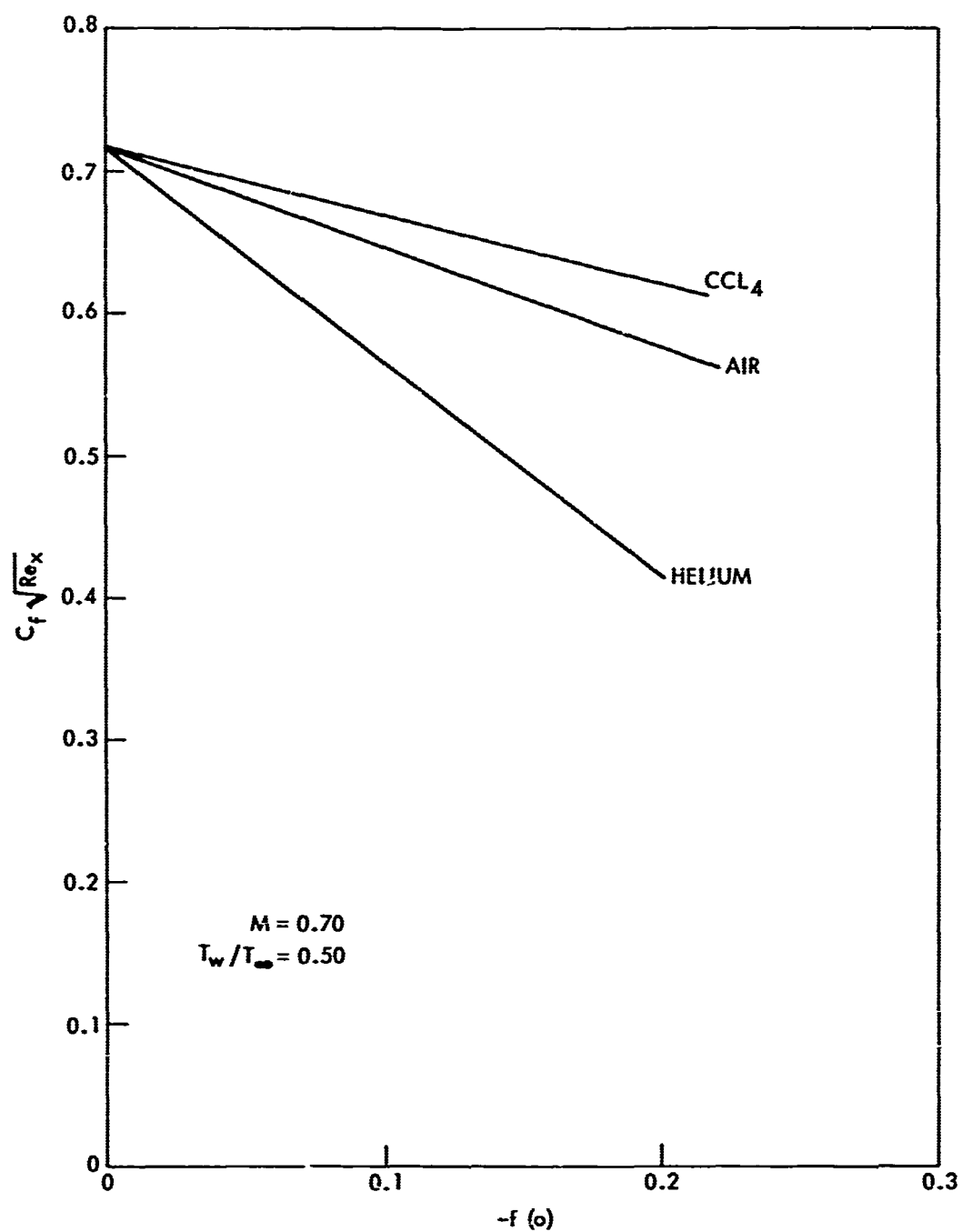
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FIG. 1 EFFECT OF INJECTION RATE ON SKIN FRICTION WITH
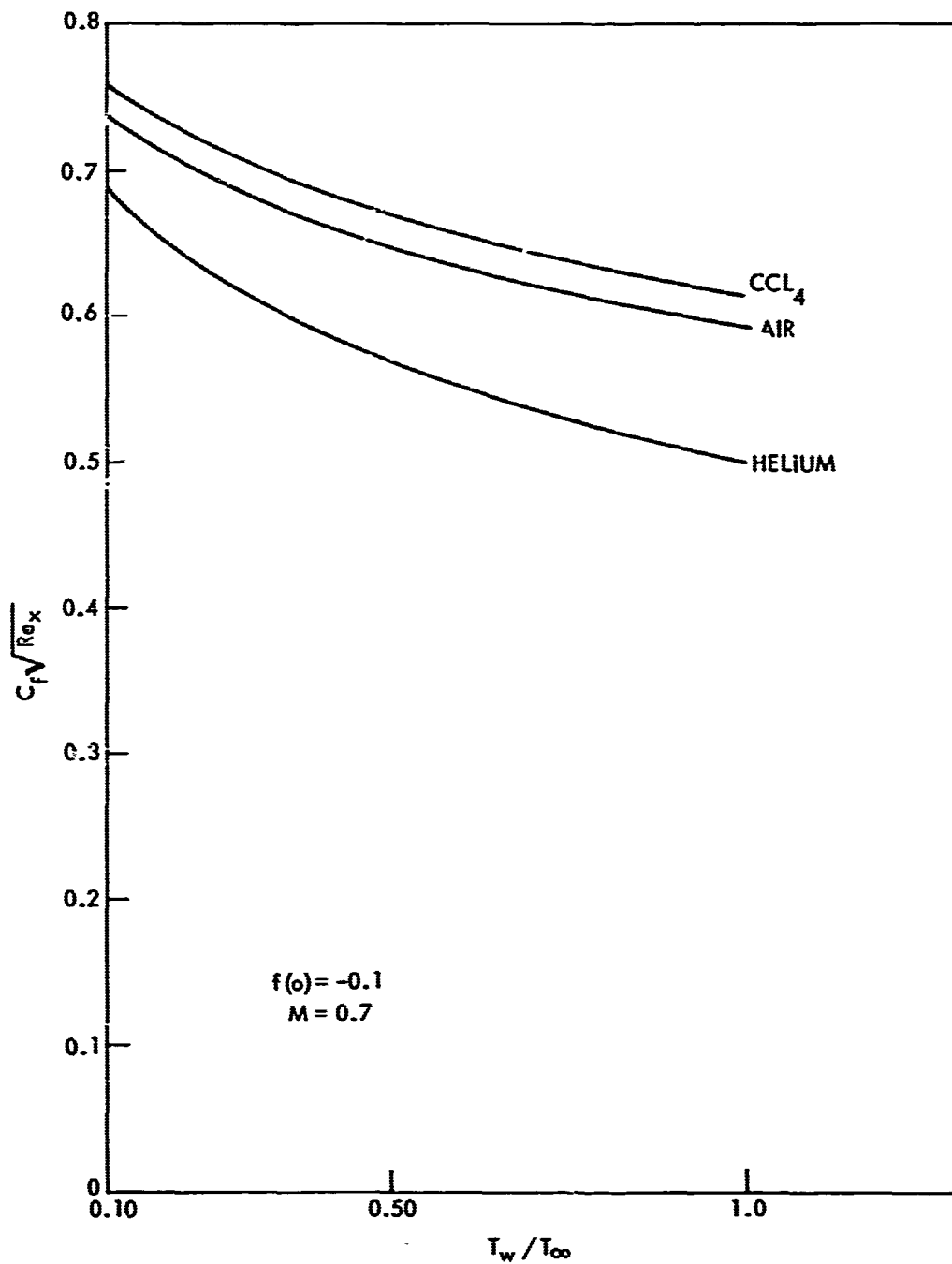LIGHT, MEDIUM, HEAVY INJECTANTS

FIG. 2 EFFECT OF WALL – STREAM TEMPERATURE RATIO ON SKIN
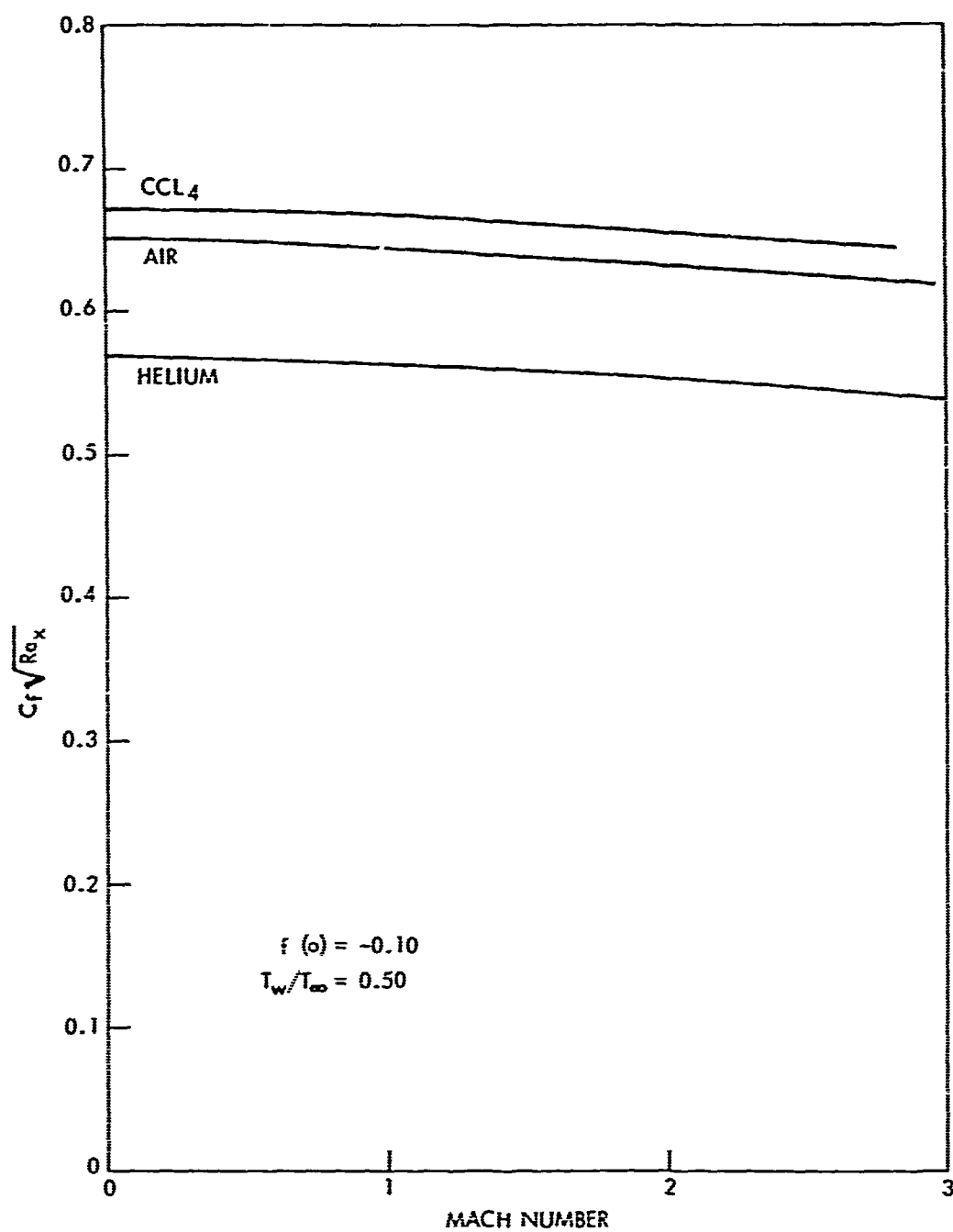FRICTION WITH LIGHT, MEDIUM, HEAVY INJECTANTS

FIG. 3 MACH NUMBER EFFECT ON SKIN FRICTION WITH LIGHT,
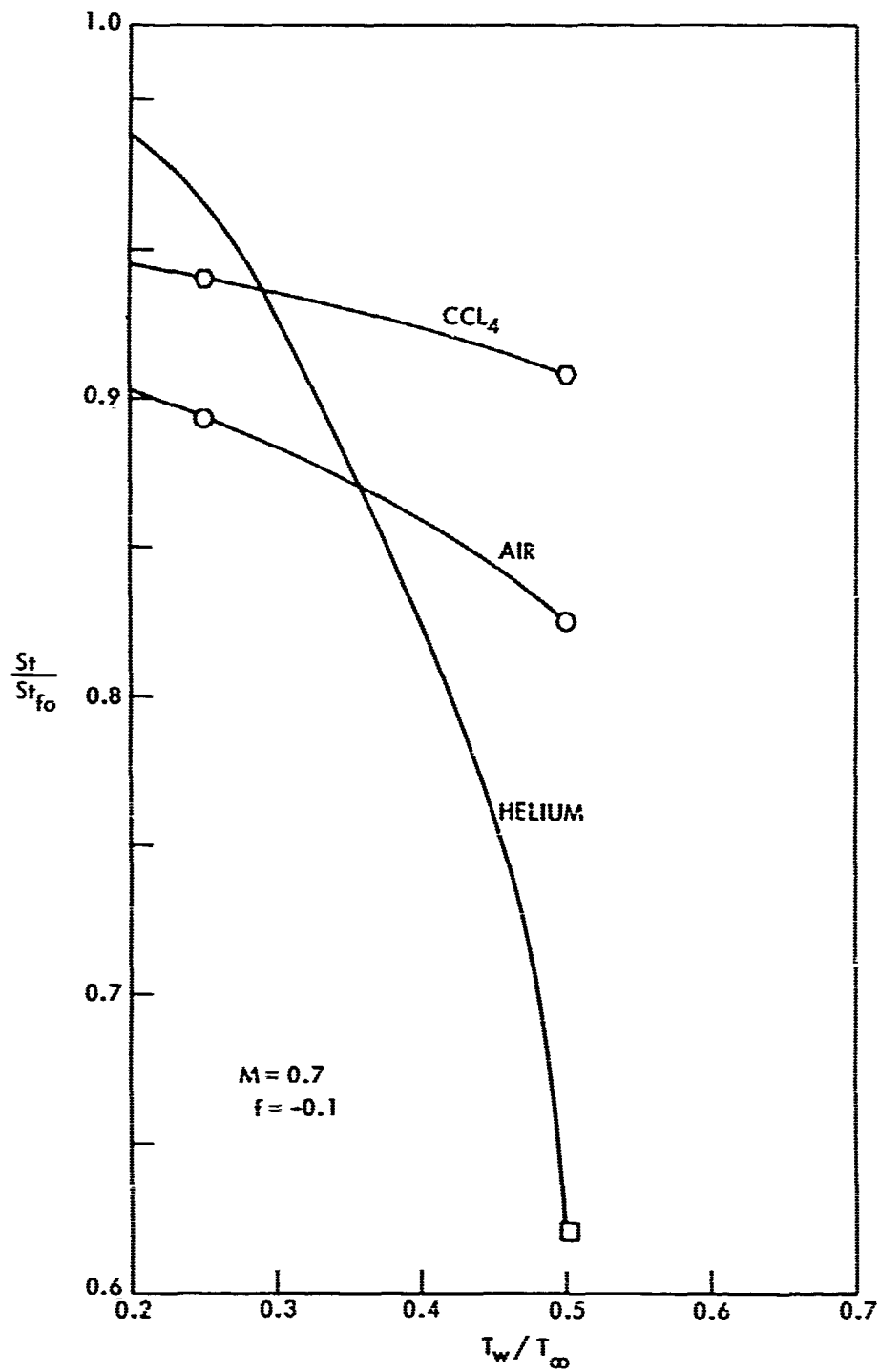MEDIUM, HEAVY INJECTANTS

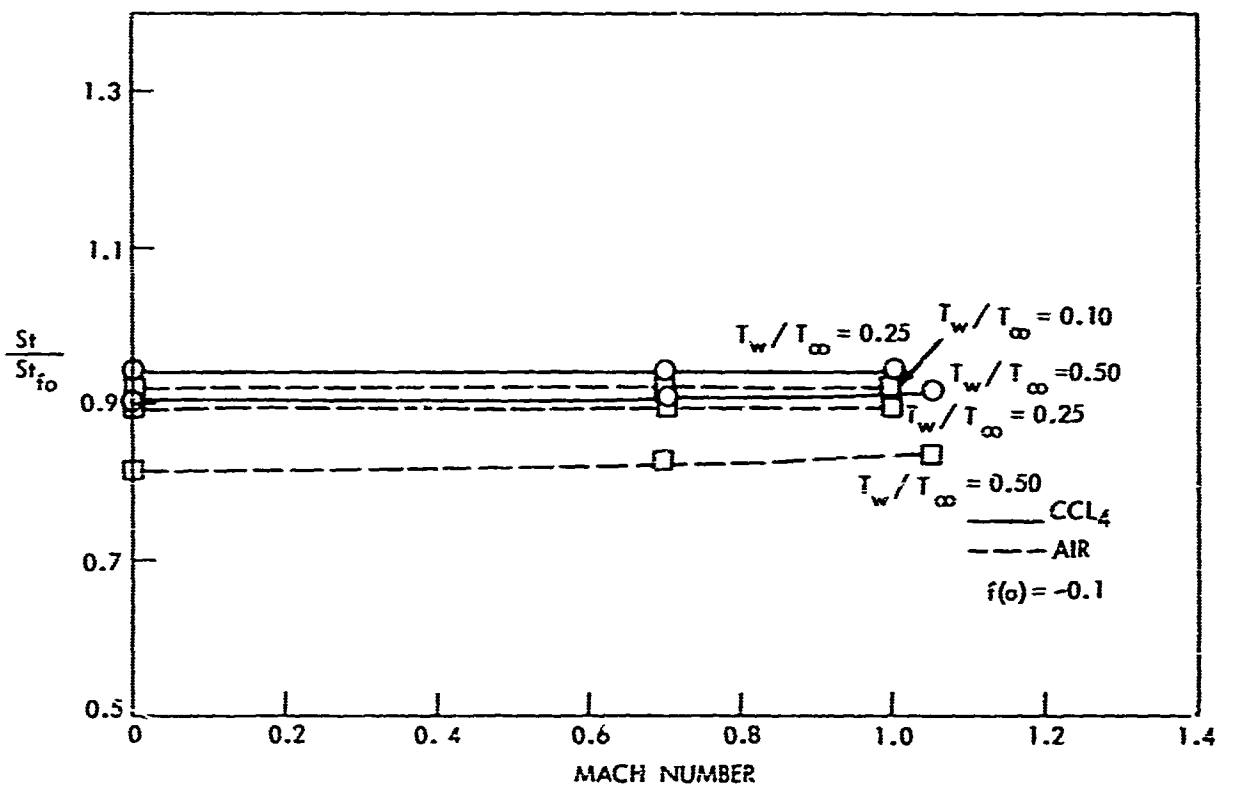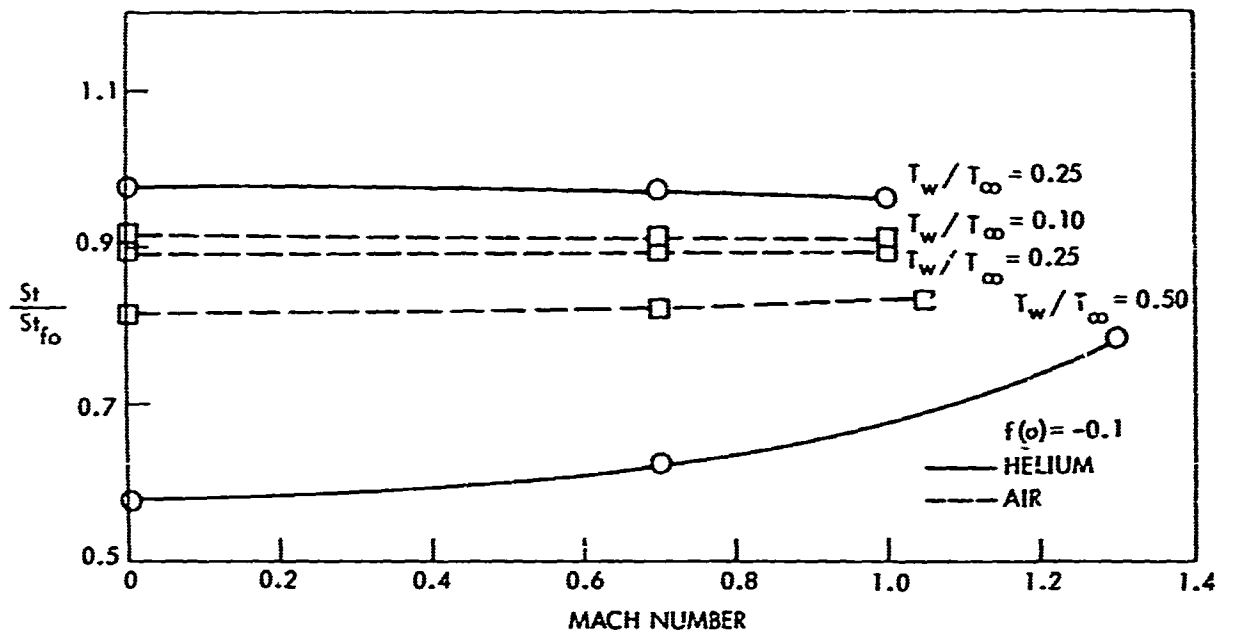FIG. 4 STANTON NUMBER RATIO VS. WALL-TO-STREAM TEMPERATURE RATIO

FIG. 5 MACH NUMBER EFFECT ON STANTON NUMBER RATIO

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rate parameter divided by the square root of the Chapman-Rubesin constant evaluated at a reference temperature (C*). This is presented in figure (6). The figure shows that the correlation worked for the Mach number range considered (Mach numbers of 0 to 3) but failed as far as the temperature ratio was concerned. Again the results predicted by equation (79) appear: whereas at a temperature ratio of 0.5, helum is the most effective coolant; when the temperature ratio is as low as 0.25, the heavier gases are more advantageous.

We have seen then that there are conditions when it is more favorable to use a heavy injected gas instead of having a light gas injection system. This is even more so when one considers the effect of injection on the stability characteristics of the boundary layer as in reference (1). Here it is shown that the heavy gas can actually cause the boundary layer to be more stable, whereas a light gas is destabilizing.

CONCLUSION

The main purpose of the investigation was to obtain boundary layer profiles showing the effects of a light, medium, and heavy gas injection. The method of solution of the governing equations has been described. The stability characteristics of the resulting profiles appear in reference (1). In addition, the effects of injection on the skin friction and heat transfer characteristics of these profiles have been obtained. The results have shown that the reduction of skin friction and heat transfer depends in general on factors which are not necessarily complimentary. For example, sometimes a large molecular diameter is advantageous, other times a small molecular weight is better, and so on. Furthermore, we have seen that the value of the wall-to-stream temperature ratio will also affect the choice of injectant. In short, the selection of discrete injection systems must represent a favorable compromise.
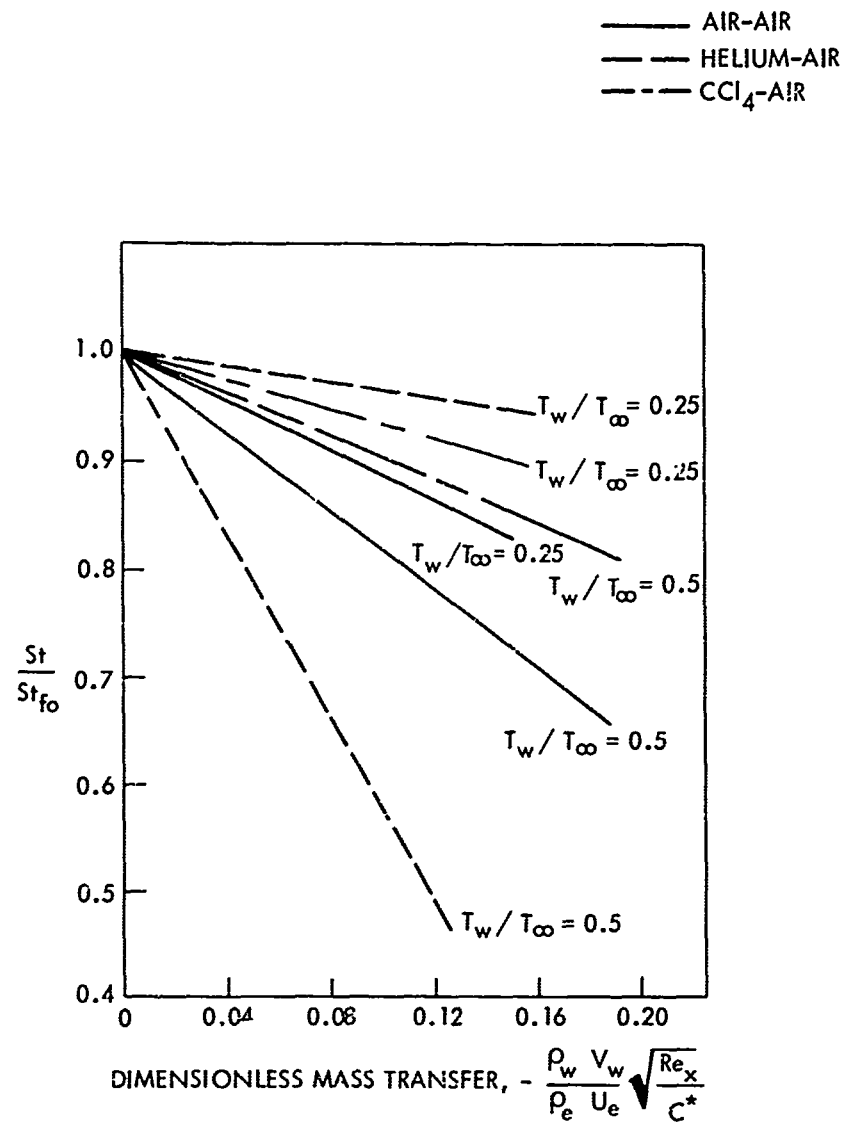
FIG. 6 EFFECT OF FOREIGN GAS INJECTION ON
HEAT TRANSFER CHARACTERISTICS

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13. ABSTRACT

    The effects of the injection of a light, medium, and heavy gas on the skin friction and heat transfer characteristics of the laminar boundary layer flow over a flat plate have been investigated. This was done by a numerical solution of the governing differential equations for injection rate parameters of -0, -0.1, -0.2; Mach numbers from 0 to 3; and wall-to-stream temperature ratios of 0.25, 0.50, and 1.00. The results show that the selection of discrete injection systems must represent a favorable compromise between conflicting factors (such as small molecular weight and large molecular diameters) which influence the amount of reduction in skin friction and heat transfer desired.

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14 KEY WORDS	LINK A		LINK B		LINK C	
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